International Journal of Theoretical Physics, Vol. 15, No. 8 (1976), pp. 617–629

High-Energy Collisions and the Breakdown of the Usual Conditions of Unitarity and Causality

JOSIP ŠOLN

Daresbury Laboratory, Warrington, WA4 4AD England
and
Harry Diamond Laboratories, Electromagnetic Effects Laboratory, Adelphi, Maryland 20783[1]

Received: 10 *October* 1975

Abstract

We discuss the questions of unitarity and causality in the ordinary and charge-conserving hadronic bremsstrahlung models, which describe quasielastic collisions of two primary particles with the emission of secondary charged pions. While the ordinary hadronic bremsstrahlung model satisfies the usual conditions of unitarity and causality, the charge-conserving hadronic bremsstrahlung model does not. The newly formulated production unitarity (which takes into account the fact that the secondary pions never occur in the initial state) is satisfied by both models. A new notion of the production causality (which seems to be a natural causality property of hadronic bremsstrahlung models) is found to be formally satisfied by both models. In view of the fact that the charge-conserving hadronic bremsstrahlung model gives a good description of the gross features of particle production whereas the ordinary model does not, we suggest that some causality requirements may in general pose overly stringent conditions on theories describing physical phenomena.

1. *Introduction and Preliminaries*

In this article we wish to investigate the extent to which the usual conditions of unitarity and causality can hold for high-energy collisions when these collisions are described by hadronic bremsstrahlung models.

In hadronic bremsstrahlung models, which describe the quasielastic collisions of two primary particles (e.g., protons) accompanied by the emission of secondary particles (e.g., pions whose energies are limited to values much less than those of protons), the S matrix is factorized as (Gemmel and Kastrup, 1969; Šoln, 1973)

$$S = S_1 S_2 \tag{1.1}$$

With S_1 and S_2 satisfying, respectively,

$$\langle \text{in}; \text{sec} | S_1 = \langle \text{in}; \text{sec} | \tag{1.2a}$$

[1] Permanent address.

$$\langle \text{in; prim} | S_2 = \sum_{n(\text{sec})} \langle 0 | S_2 | \text{in}; n(\text{sec})\rangle \langle \text{in}; n(\text{sec}); \text{prim} | \qquad (1.2\text{b})$$

where $|\text{prim; in}\rangle$ and $|\text{sec; in}\rangle$ are any "in" states of primary and secondary particles, respectively, the summation goes over a complete set of "in" secondary particle states, and we do not demand that the S_2 matrix be translationally invariant. Relations (1.2a) and (1.2b) mean that the S_1 and S_2 matrices can be expanded in terms of free field "in" operators of primaries and secondaries, respectively. It is customary to ignore the spin and isospin of primaries and often the isospin and parity of secondaries. From (1.1), (1.2a), and (1.2b) it is easy to see that as far as conditions of unitarity and causality are concerned, the S_1 and S_2 matrices can be discussed separately. In what follows we shall assume that the S_1 matrix satisfies the usual conditions of unitarity and causality (Bogoliubov and Shirkov, 1959).

It is convenient to associate with S_2 the inelastic coupling constant g and write

$$S_2(g) = S_2{}^0(g)R_2(g), \qquad S_2{}^0(g) = \langle 0 | S_2(g) | 0\rangle \qquad (1.3)$$

where we require

$$S_2{}^0(0) = 1, \qquad R_2(0) = 1 \qquad (1.4)$$

Denoting the initial and final momenta of primaries by p_i, p_i' $(i = 1,2)$ and the final momenta of secondaries by k_i $(i = 1, \ldots, n)$, then according to (1.1), (1.2a), (1.2b), and (1.3), we have

$$\langle k_1, \ldots, k_n; p_1', p_2' | S | p_1, p_2\rangle = \langle p_1', p_2' | S_1 | p_1, p_2\rangle S_2{}^0(g) \langle k_1, \ldots, k_n | R_2(g) | 0\rangle \qquad (1.5)$$

We can write

$$\langle p_1', p_2' | S_1 | p_1, p_2\rangle = M_1(s, t)\delta^{(4)}(p_1 + p_2 - p_1' - p_2') \qquad (1.6)$$

where s is the c.m. (center of mass) energy squared and $(-t)$ is the invariant momentum transfer squared of primaries. From (1.5) we can interpret $S_2{}^0(g)\langle k_1, \ldots, k_n | R_2(g) | 0\rangle$ as the conditional (probability) transition amplitude for the emission of n secondaries subject to the hypothesis of the occurrence of the "primary" process $p_1 + p_2 \to p_1' + p_2'$. Consequently $\langle k_1, \ldots, k_n | S_2(g) | 0\rangle$ will depend on $\langle p_1', p_2' | S_1 | p_1, p_2\rangle$ or equivalently on s and t [see relation (1.6)]. One calls $\langle p_1', p_2' | S_1 | p_1, p_2\rangle$ a "potential" transition amplitude since, according to (1.4), it is the only existing amplitude in the limit $g \to 0$ (Gemmel and Kastrup, 1969).

There is one thing that is prominent for any hadronic bremsstrahlung model with assumptions (1.1), (1.2a), and (1.2b): the division of particles into primaries and secondaries. As we can see from (1.5), primaries can occur in both the initial and final states, while secondaries will occur only in the final state. Thus the initial state is the vacuum as far as secondaries are concerned. This fact will be taken into account when we discuss the unitarity and the causality of the S_2 matrix.

Let us now assume that we are interested in the production of secondary charged pions (say in pp collisions) and that it is of no concern to us what the charges of individual pions are. Then we can describe such a system with a single field operator, $\phi_{\text{in}}(x)$, which we can call a charge number field operator (Šoln, 1974a, b). One can discuss the production of secondary charged pions within the framework of two hadronic bremsstrahlung models: the ordinary hadronic bremsstrahlung model in which the charge and parity conservations are ignored and the charge-conserving hadronic bremsstrahlung model, which, in addition to charge conservation, obeys also the parity conservation (Gemmel and Kastrup, 1969; Šoln, 1973, 1974a, 1974b). The S_2 matrix for the ordinary hadronic bremsstrahlung model we denote with S_2' and write (Šoln, 1974a, b)

$$\begin{aligned} S_2'(g) &= S_2'^0(g)R_2'(g) \\ S_2'^0(g) &= \exp i\frac{g^2}{2}\int d^4x d^4y j(x)\Delta_F(x-y)j(y) \\ R_2'(g) &= :\exp ig\int d^4x j(x)\phi_{\text{in}}(x): \end{aligned} \tag{1.7}$$

where $j(x)$ is the real c-number source of secondaries due to primaries [see the discussion after relation (1.6)] and $\Delta_F(x-y)$ is the Feynman Green's function of secondaries. For the charge-conserving hadronic bremsstrahlung model, the S_2 matrix (denoted as S_2'') is given as (Šoln, 1974a, b)

$$\begin{aligned} S_2''(g) &= S_2''^0(g)R_2''(g) \\ S_2''^0(g) &= [\cosh ig^2\int d^4x d^4y \omega^*(x)\Delta_+(x-y)\omega(y)]^{-1/2} \\ R_2''(g) &= :\cosh g\int d^4x \omega(x)\phi_{\text{in}}(x): \end{aligned} \tag{1.8}$$

where $\omega(x)$ is generally a complex c-number source of charged secondary pions due to primaries and $\Delta_+(x-y)$ is the positive frequency Δ function satisfying the homogeneous Klein-Gordon equation in which the mass is that of the secondary particle. Let us also point out that in view of the discussion after relation (1.6), both $j(x)$ from (1.7) and $\omega(x)$ from (1.8) depend in principle on s and t [see relation (1.6)]. As we shall see in detail later the S_2' matrix from (1.7) satisfies the usual conditions of unitarity and causality while the S_2'' matrix from (1.8) does not. However, the S_2'' matrix will satisfy what we shall call the production unitarity and the production causality.

In order to be able to discuss briefly the question of charge conservation, we define the number operator for the charged secondary pions as

$$N = \int \frac{d^3k}{E(k)} a^+(k)a(k) \tag{1.9}$$

where $a(k)$ and $a^+(k)$ are the annihilation and creation operators associated with the charge number field operator $\phi_{\text{in}}(x)$. With the help of N we define a new operator U as

$$U = \exp[-i\pi N] \tag{1.10}$$

Now it is not difficult to see that the charge conserving S_2 matrix must satisfy

$$US_2(g)U^{-1} = S_2(g) \tag{1.11}$$

since the charge conservation demands that the initial and final pion states give the same eigenvalues for U. As we can see from (1.7), $US_2'(g)U^{-1} = S_2'^{+}(g)$. Thus the ordinary hadronic bremsstrahlung model when applied to the production of charged pions (say in pp collisions) violates the charge conservation. However, from (1.8) we easily see that $US_2''(g)U^{-1} = S_2''(g)$, and indeed the charge-conserving hadronic bremsstrahlung model conserves the charge in the production of charged secondary pions. Taking into account that the c-number sources $j(x)$ and $\omega(x)$ can only be scalars, we see that under parity transformation $[\phi_{\text{in}}(\mathbf{x}, x^4) \to -\phi_{\text{in}}(-\mathbf{x}, x^4)]$, $S_2'(g) \to S_2'^{+}(g)$ and $S_2''(g) \to S_2''(g)$. Consequently, we do not have parity conservation in the ordinary hadronic bremsstrahlung model while in the charge conserving hadronic bremsstrahlung model we do. It is quite clear that from the practical point of view the S_2'' matrix is more attractive than the S_2' matrix in discussing the production of charged secondary pions, particularly in view of the fact that the S_2'' matrix gives at once a very good agreement with experiments in the so-called lower-energy regime (Soln, 1974a, b) for the multiplicity distribution functions for secondary charged pions (Šoln, 1973, 1974a, b) and the correlation parameters for the secondary negative pions (Šoln, 1974a, b, 1975). Since S_2'' violates the usual conditions of unitarity and causality, we inquire whether these conditions are too strong for high-energy collisions and can be replaced with weaker ones.

2. *Production Unitarity*

As is known, in general the S matrix elements are probability amplitudes to find the system at $t \to \infty$ in some state $|\,\text{out}; b\rangle$ when some state $|\,\text{in}; \alpha\rangle$ at $t \to -\infty$ has been given. More specifically

$$|\,\text{in}; \alpha\rangle = \sum_{(b)} |\,\text{out}; b\rangle\langle\text{in}; b\,|\,S\,|\,\text{in}; \alpha\rangle \tag{2.1}$$

where the common indices α and b specify states according to a complete set of commuting observables. The differences between states $|\,\text{in}; \alpha\rangle$ and $|\,\text{in}; a\rangle$ (or $|\,\text{in}; b\rangle$) is that

$$\sum_{(a)} |\,\text{in}; a\rangle\langle\text{in}; a\,| = 1 \tag{2.2}$$

while

$$\sum_{(\alpha)} |\,\text{in}; \alpha\rangle\langle\text{in}; \alpha\,| = P \tag{2.3}$$

where P is essentially the projection operator on some subspace of the whole Hilbert space. In other words, while the states denoted as $|\,\text{in}; a\rangle$ or $|\,\text{in}; b\rangle$ are complete, the states denoted as $|\,\text{in}; \alpha\rangle$ or $|\,\text{in}; \beta\rangle$ are not, and in the formal limit $\alpha \to a$, P from (2.3) goes into 1. Of course we do demand the orthonormality of states $|\text{in};\alpha\rangle$; thus from (2.1) we have

$$\langle \text{in};\beta|\text{in};\alpha\rangle = \sum_{(b)} \langle \text{in};\beta|S^+|\text{in};b\rangle\langle \text{in};b|S|\text{in};\alpha\rangle$$
$$= \langle \text{in};\beta|S^+S|\text{in};\alpha\rangle \tag{2.4}$$

which is what we call production unitarity. From the physical point of view, the projection operator P allows only those states that are physically accessible as initial states at $t \to -\infty$. Since $P|\text{in};\alpha\rangle = |\text{in};\alpha\rangle$, $P|\text{in};\beta\rangle = |\text{in};\beta\rangle$, we can also write $\langle \text{in};\beta|S^+S|\text{in};\alpha\rangle$ in (2.4) as $\langle \text{in};\beta|PS^+SP|\text{in};\alpha\rangle$. Consequently we can define a new S matrix (denoted by Σ) as

$$\Sigma = SP \tag{2.5}$$

for which the production unitarity can be written as

$$\Sigma^+\Sigma = PS^+SP = P \tag{2.6}$$

The advantage of (2.6) over (2.4) is that (2.6) formally is valid in the whole Hilbert space. It is clear that we could have started immediately the whole formulation of the production unitarity in terms of Σ, writing instead of (2.1)

$$|\text{in};\alpha\rangle = \sum_{(b)} |\text{out};b\rangle\langle \text{in};b|\Sigma|\text{in};\alpha\rangle \tag{2.1$'$}$$

where obviously

$$|\text{out};b\rangle = \Sigma^+|\text{in};b\rangle \equiv PS^+|\text{in};b\rangle \tag{2.7}$$

which is consistent with $|\text{in};\alpha\rangle \equiv P|\text{in};\alpha\rangle$. Clearly, relation (2.7) allows us to discuss only matrix elements of the type $\langle \text{in};\alpha|\text{out};b\rangle$ which reduces to $\langle \text{in};\alpha|S^+|\text{in};b\rangle$.

It is quite clear that if the S matrix satisfies the usual condition of unitarity $S^+S = 1$, then production unitarity (2.6) for the Σ matrix is automatically satisfied.

Let us now apply this general type of formalism to our hadronic bremsstrahlung models. In the discussion, we will concentrate mostly on S_2 matrices, since we are assuming that, in principle, S_1 matrices satisfy usual unitarity and causality conditions.

The S_2' matrix of the ordinary hadronic bremsstrahlung model [see (1.7)] satisfies the usual unitarity condition

$$S_2'^+(g)S_2'(g) = 1 \tag{2.8}$$

Let us demonstrate this. First, from $S_2'(g)$ we define $S_2'(g;\eta(x))$ by changing $\phi_{\text{in}}(x)$ to $\phi_{\text{in}}(x) + \eta(x)$ in (1.7):

$$S_2'(g;\eta(x)) = S_2'(g)\exp ig\int d^4x j(x)\eta(x) \tag{2.9}$$

Now (2.8) requires that

$$\langle k_{r+1},\ldots,k_n|S_2'^+(g)S_2'(g)|k_1,\ldots,k_r\rangle = \langle k_{r+1},\ldots,k_n|k_1,\ldots,k_r\rangle \tag{2.10}$$

from which it is not difficult to deduce that in general (for details of this kind of calculation see reference Bogoliubov and Shirkov (1959))

$$\frac{\delta^l}{\delta\eta(x_1)\cdots\delta\eta(x_l)}\langle 0|S_2'^{+}(g;\eta(x))S_2'(g;\eta(x))|0\rangle\Big|_{\eta=0}=0,\ l=1,\ldots,n \quad (2.11)$$

Since n is arbitrary, so is l, and consequently from (2.11) we conclude that $\langle 0|S_2'^{+}(g;\eta)S_2'(g;\eta)|0\rangle$ must be independent of η. Since in general $S_2'(g,\eta=0) = S_2'(g)$, we have that the condition of usual unitarity reduces to

$$\langle 0|S_2'^{+}(g;\eta(x))S_2'(g;\eta(x))|0\rangle=\langle 0|S_2'^{+}(g)S_2'(g)|0\rangle=1 \quad (2.12)$$

From (2.9) we see that (2.12) is indeed satisfied provided that $\langle 0|R_2'^{+}(g)R_2'(g)|0\rangle = |S_2'^{0}(g)|^{-2}$, which can be easily verified from (1.7). Let us point out that in deriving the equivalence of (2.12) with (2.8) at no place did we use explicitly the properties of the ordinary hadronic bremsstrahlung model. Thus the equivalence of (2.12) to (2.8) holds generally, as long as we talk about self-interacting bosons.

As mentioned earlier, in hadronic bremsstrahlung models there are not secondaries in the initial state (at $t\to-\infty$) and consequently

$$P_2=|0\rangle\langle 0| \quad (2.13)$$

Thus for the ordinary hadronic bremsstrahlung model

$$\Sigma_2'(g)=S_2'(g)|0\rangle\langle 0| \quad (2.14)$$

and indeed $\Sigma_2'^{+}\Sigma_2' = |0\rangle\langle 0|$.

Concerning the charge-conserving hadronic bremsstrahlung model, it is not difficult to see that [see (1.8)] $S_2''^{+}(g)\,S_2''(g)\neq 1$; i.e., the usual unitarity condition is not satisfied. However, production unitarity is satisfied:

$$\begin{aligned}\Sigma_2''(g)&=S_2''(g)|0\rangle\langle 0|\\ \Sigma_2''^{+}(g)\Sigma_2''(g)&=|0\rangle\langle 0|S_2''^{+}(g)S_2''(g)|0\rangle\langle 0|=|0\rangle\langle 0|\end{aligned} \quad (2.15)$$

provided that $\langle 0|R_2''^{+}(g)R_2''(g)|0\rangle = |S_2''(g)|^{-2}$, which can be directly verified. Thus in either case of the ordinary or the charge-conserving hadronic bremsstrahlung model, we have the same production unitarity.

3. *Production Causality*

In discussions of the usual conditions of causality on the S matrix, it is useful to introduce the notion of the space-time interaction region G (Bogoliubov and Shirkov, 1959), Such a notion is particularly applicable for strong interactions, where the interaction is confined in a volume of the order of magnitude of 1 fm^3 and it lasts for about 10^{-23} sec. In the usual formulation of the condition of causality, one defines an interaction space-time region G by assuming the coupling constant g formally to depend on x and as such to be different from zero only in G. Next one divides region G into two separate subregions, G_1 and G_2, in such a way that all points of subregion G_1 lie in the past with respect to a certain time instant, t, while all points of subregion G_2 lie in the future with respect to t. This one formally denotes as $G_2 > G_1$. Furthermore, writing $g(x)$ as

$$g(x) = g_1(x) + g_2(x) \tag{3.1}$$

where $g_1(x)$ and $g_2(x)$ differ from zero only in G_1 and G_2, respectively, the usual condition of causality is then expressed as (Bogoliubov and Shirkov, 1959).

$$S(g_1 + g_2) = S(g_2)S(g_1) \qquad \text{for } G_2 > G_1 \tag{3.2}$$

There is no doubt that (3.2) describes the spirit of the causality in the sense that any event occurring in the system may exert an influence on the evolution of the system only in the future.

Condition of causality (3.2) is quite strong as far as the conservation laws are concerned: It demands that in each subregion—no matter how many of them we wish to visualize—the conservation law must hold if we wish it to hold in the whole space-time interaction region G, which is the only thing that we can actually verify experimentally. More specifically, in the case of charge conservation, (3.2) demands that when charged particles are produced, at least 2 of them (with total zero charge) must be produced from the same space-time point x, not both of them necessarily being real particles (for example, one could be a virtual particle that can travel to another region where the second real charged particle is produced). While, on the one hand, this is quite appealing as far as the charge conservation law goes, on the other hand, it also means that one cannot associate the creation of a single charged particle with a single subregion. However, to us it looks perfectly causal if in region G_1, first, say, a negative particle is produced which now causes the production of a positive particle in region G_2, so that the end result is the emergence of a positive-negative pair from the whole region G.

By assuming the unitarity $S^+(g)S(g) = 1$ from (3.2) one can derive the causality condition in the "differential form"

$$\frac{\delta}{\delta g(y)}\left[S^+ \frac{\delta S}{\delta g(x)}\right] = 0 \qquad \text{for } x \lesssim y \tag{3.3}$$

where $x \lesssim y$ means $x^4 < y^4$ and $(x - y)^2 > 0$. The interpretation of (3.3) is quite simple. The quantity $S^+\delta S/\delta g(x)$ cannot receive "communications" from points y which are either a spacelike distance from x or later than x. Instead of (3.3) one quite often writes the condition of causality in the "differential form" as

$$\frac{\delta}{\delta \phi_{\text{in}}(y)}\left[S^+ \frac{\delta S}{\delta \phi_{\text{in}}(x)}\right] = 0 \qquad \text{for } x \lesssim y \tag{3.4}$$

where now for (3.4) we can make the interpretation that the quantity $S^+\delta S/\delta\phi_{\text{in}}(x)$ cannot receive "communications" from a particle [associated with $\phi_{\text{in}}(y)$] at space-time point y for which $x^4 < y^4$ and $(x - y)^2 > 0$.

In hadronic bremsstrahlung models we can think of colliding primary particles providing a space-time interaction region from which secondary particles emerge. As we already mentioned, the only accessible initial state of secondaries is a vacuum. As far as the ordinary hadronic bremsstrahlung model is concerned, however, one may assume any secondary particle state as an initial state at $t \to -\infty$ (although physically we always talk of a vacuum as an initial state). Consequently, we expect S_2' matrix (1.7) to obey causality conditions (3.2), (3.3), and (3.4). This indeed can be easily verified by writing (1.7) in equivalent form

$$S_2'(g) = T \exp ig \int d^4x j(x) \phi_{\text{in}}(x) \tag{3.5}$$

In order to verify (3.2) and (3.3), one pulls g under the sign of the integral and writes it as $g(x)$. Then by virtue of the time ordering operator T, one clearly has

$$\begin{aligned} &T \exp i \int d^4x [g_2(x) + g_1(x)]\, j(x) \phi_{\text{in}}(x) \\ &= T[\exp i \int d^4x g_2(x) j(x) \phi_{\text{in}}(x)]\; T[\exp i \int d^4y g_1(y) j(y) \phi_{\text{in}}(y)] \\ &\qquad \text{for } G_2 > G_1 \end{aligned} \tag{3.6}$$

and

$$\frac{\delta}{\delta g(y)} \left[S_2'^{+} \frac{\delta S_2'}{\delta g(x)} \right] = 0 \text{ for } x \lesssim y \tag{3.7}$$

Similarly one shows that

$$\frac{\delta}{\delta \phi_{\text{in}}(y)} \left[S_2'^{+} \frac{\delta S_2'}{\delta \phi_{\text{in}}(x)} \right] = 0 \text{ for any } x \text{ and } y \tag{3.8}$$

We can now take into account that as far as the secondaries are concerned, the initial state at $t \to -\infty$ is a vacuum. Thus instead of (3.6) and (3.7) we can now write [see (2.14)]

$$\Sigma_2'(g_1 + g_2) = S_2'(g_2) \Sigma_2'(g_1), \text{ for } G_2 > G_1$$

$$\frac{\delta}{\delta g(y)} \left[S_2'^{+} \frac{\delta \Sigma_2'}{\delta g(x)} \right] = 0 \text{ for } x \lesssim y$$

However, it does not appear to be a simple matter to rewrite (3.8) in such a way as to involve $\Sigma_2' = S_2' P_2 (P_2 = |0\rangle\langle 0|)$. This is due to the fact that (for $P_2 \neq 1$) $[\phi_{\text{in}}(x), P_2] \neq 0$, $\delta P_2 / \delta \phi_{\text{in}}(x) \neq 0$. Consequently we conclude that once the number of accessible initial states at $t \to -\infty$ is restricted, causality conditions like (3.4) may be very difficult to formulate even if we have formally $S^+ S = SS^+ = 1$

Let us finally discuss the causality of the charge-conserving hadronic bremsstrahlung model. This model is of particular interest, since, unlike the ordinary hadronic bremsstrahlung model, it obeys the parity and charge conservation laws when the charged secondary pions are described by the charge number field operator $\phi_{in}(x)$. $S_2''(g)$ matrix (1.8), which describes the production of secondary charged pions, obeys only what we call production unitarity (2.15). Since causality conditions (3.3) and (3.4) rely on $S^+S = SS^+ = 1$, we see that S_2'' cannot satisfy them. Nevertheless, we wish to see if at least formally the $S_2''(g)$ matrix satisfies some kind of causality, particularly in view of the fact that it does satisfy what we call the "macroscopic causality" $S_2''(g = 0) = 1$ (Šoln, 1965). We notice that (1.8) can be rewritten as

$$
\begin{aligned}
S_2''(g) &= F(g)Q(g) \\
F(g) &= [\cosh ig^2 \int d^4x d^4x' \omega^*(x)\Delta_+(x - x')\omega(x')]^{-1/2} \left[\exp \frac{ig^2}{2} \int d^4x d^4x' \times \right. \\
&\quad \left. \times\, \omega(x)\Delta_F(x - x')\omega(x')\right] \qquad (3.9) \\
Q(g) &= T \cosh g \int d^4x \omega(x)\phi_{in}(x)
\end{aligned}
$$

As we see, (3.9) gives the S_2'' matrix as a product of c number $F(g)$ and q number $Q(g)$ which is properly time ordered by means of the T operator. Now what we are really interested in is $\Sigma_2''(g) = S_2''(g)P_2 (P_2 = |0\rangle\langle 0|)$, see (2.15), which is given now as

$$\Sigma_2''(g) = F(g)Q(g)|0\rangle\langle 0|$$

The transition amplitude from a vacuum to $2m$ charged pions is (with k_i, $i = 1, \ldots, 2m$, denoting their momenta)

$$
\begin{aligned}
\langle k_1, \ldots, k_{2m} | \Sigma_2''(g) | 0 \rangle &= \langle k_1, \ldots, k_{2m} | S_2''(g) | 0 \rangle \\
&= F(g)\langle k_1, \ldots, k_{2m} | Q(g) | 0 \rangle \qquad (3.10)
\end{aligned}
$$

From (3.10) we see that $F(g)$ is the c-number multiplicative factor appearing in every matrix element which is essentially determined by $Q(g)$. It is not difficult to see that the main role of $F(g)$ is to make the $\Sigma_2''(g)$ matrix obey production unitarity (2.15). It is quite clear that in view of the time-ordering operator T, $Q(g)$ from (3.9) is causal at least on the formal level. Namely, in the matrix element (3.10), the "communications" from a particle associated with $\phi_{in}(y)$ cannot be received by a particle associated with $\phi_{in}(x)$ if $x \lesssim y$. We can also demonstrate the formal causality of $Q(g)$ by pulling g's inside integrals and making them x dependent. Then (as at the beginning of this section) we divide the whole space-time interaction region G into the subregions, G_1 and G_2 ($G_2 > G_1$), where the space-time-dependent coupling constant $g(x) = g_1(x) + g_2(x)$, with $g_1(x) \neq 0$ in G_1 and $g_2(x) \neq 0$ in G_2. Then from (3.9) we have

$$
\begin{aligned}
Q(g_1 + g_2) &= T \cosh \int d^4x g(x)\omega(x)\phi_{in}(x) \\
&= Q(g_2)Q(g_1) + [T \sinh \int d^4x g_2(x)\omega(x)\phi_{in}(x)] \times \\
&\quad \times [T \sinh \int d^4x g_1(x)\omega(x)\phi_{in}(x)] \text{ for } G_2 > G_1 \qquad (3.11)
\end{aligned}
$$

Relation (3.11) looks almost like (3.2) except that it has additional terms with sinh functions. Similarly one can divide the interaction space-time region G into any number of subregions, $G_1, \ldots, G_n$ ($G_n > G_{n-1} > \cdots > G_1$) and again one would show that the formal causality of $Q(g)$ holds.

This type of causality, where the S matrix is causal up to a c-number factor relations (3.9)-(3.11), we shall call the production causality. This form of causality, like many other different forms of causality, may only be formally satisfied for specific situations. Namely, the concept of the production causality for the $S''_2(g)$ matrix really only enters through the $Q(g)$ matrix. Now if some $Q(g)$ matrix elements are infinite, then the production causality has only a formal meaning since the concept of "communications" between particles becomes unclear.

Before we analyze the causality properties of the $S''_2(g)$ matrix, let us note that in the matrix elements the pions are always on the mass shell ($k_i^2 = -\mu_\pi^2$, $i = 1,2 \ldots$; μ_π is the mass of the charged secondary pions). Now, when one calculates the multiplicity distribution function for a secondary charged pion, the only integral that we need to evaluate is

$$\int d^4x d^4y \omega^*(x)\Delta_+(x-y)\omega(y) = \frac{1}{2i(2\pi)^3}\int \frac{d^3k}{E(k)} \,|\tilde{\omega}(k)|^2$$

$$E(k) = (\mathbf{k}^2 + \mu_\pi^2) \tag{3.12}$$

where $\tilde{\omega}(k)$ is the Fourier transform of $\omega(x)$. Gemmel and Kastrup (1969) give an example of $\omega(x)$ within the ordinary hadronic bremsstrahlung model [$\omega(x) = ij(x)$, $j(x)$ real] for which $\tilde{\omega}(k)$ is finite for $k^2 = -\mu_\pi^2$ and for which the integral (3.12) is finite. Consequently, we shall assume that also in the case of the charge-conserving hadronic bremsstrahlung model the "pion source" function $\omega(x)$ can be chosen in such a way that $\tilde{\omega}(k)$ is finite and also that the integral of the type (3.12) can be made finite.

The situation involving the production causality is quite different, since now we will have to get involved with $\tilde{\omega}(k)$ also for $k^2 \neq -\mu_\pi^2$ (pion momentum off the mass shell). Namely, from (3.9) we see that for the $S''_2(g)$ matrix to contain the time-ordering T operator, we acquired a new factor

$$\exp\frac{ig^2}{2}\int d^4x d^4x' \omega(x)\Delta_F(x-x')\omega(x')$$

$$= \exp\frac{ig^2}{2}\left[\frac{1}{2i(2\pi)^3}\int\frac{d^3k}{E(k)}\tilde{\omega}(-k)\tilde{\omega}(k) + \frac{1}{(2\pi)^4}\int d^4k \frac{P}{k^2+\mu_\pi^2}\tilde{\omega}(-k)\tilde{\omega}(k)\right] \tag{3.13}$$

where the symbol P excludes those k's from the integral for which $k^2 + \mu_\pi^2 = 0$. Factor (3.13) is unobservable in the $S''_2(g)$ matrix elements, for it is canceled by the inverse of the same factor contained in $Q(g)$ [compare (3.9) with (1.8)]. Consequently, if we wish to have $Q(g)$ truly causal, factor (3.13) should be finite. Now, in the first term of (3.13) the integral is performed exclusively over the pion mass shell, and we shall assume that it can be made finite. In the second term of (3.13) the integral is performed exclusively off the pion mass-shell, and it is this term that we wish to investigate more closely. First of all, we can write

$$\frac{P}{k^2+\mu_\pi^2}=\frac{1}{2}\left\{\frac{1}{[E(k)-k^4-i\epsilon]\,[E(k)+k^4-i\epsilon]}+\frac{1}{[E(k)-k^4+i\epsilon]\,[E(k)+k^4+i\epsilon]}\right\}\ \epsilon\to+0 \quad (3.14)$$

We see that as far as (3.14) is concerned, the integral along the real axis of k^4 can be appropriately distorted and thus avoids the singularities at $k^4=\pm E(k)$. However, if $\tilde{\omega}^*(k)=\pm\tilde{\omega}(-k)$ [meaning that $\omega(x)$ is either real or purely imaginary], then it is irrelevant what the singularity structure of $\tilde{\omega}(k)$ is, for the second integral in (3.13) is real and as such contributes to (3.13) only a phase factor which is always finite. When $\tilde{\omega}(x)$ is complex, one has to make sure that the imaginary part of the second integral in (3.13) is finite. This can be achieved only if the singularity structure of $\tilde{\omega}(k)$ in variable k^4 does not cause pinching singularities, thus making the contour free for distortion.

Finally, let us discuss the question of conservation laws. As far as the whole space-time interaction region G is concerned, the Σ_2'' matrix obeys the charge and parity conservation laws {one can easily show, for example, that $[U,\Sigma_2'']=0$, where U is defined by (1.10)}. As a matter of fact, experimentally we can only observe conservation laws for the whole region G. In fact, (3.11) clearly shows that in the whole region G both charge and parity are conserved. However, in each of the subregions G_1 and G_2, separately, neither parity nor charge is entirely conserved. Namely, while $Q(g_1)$ and $Q(g_2)$ conserve charge and parity in G_1 and G_2, respectively,

$$T\sinh\int d^4x g_1(x)\omega(x)\phi_{\text{in}}(x)$$

and

$$T\sinh\int d^4x g_2(x)\omega(x)\phi_{\text{in}}(x)$$

violate both charge and parity conservation in G_1 and G_2, respectively. This, of course, is unobservable, and it should not pose a particular concern. Actually, here we see the spirit of causality: As soon as an odd number of charged secondary mesons are created within the subregion G_1, they cause the creation of another odd number of secondary charged mesons within the remaining subregion G_2, so that the total number of emitted charged secondary mesons from the total space-time region G is even. Since the created secondary charged pions within region G_1 are not constant for the fixed even number of produced secondary charged pions from region G, we see that some secondary particles from region G_1 may actually be absorbed by region G_2.

4. *Discussion and Conclusion*

We have shown, at least as far as the hadronic bremsstrahlung models are concerned, that the usual concepts of unitarity and causality need not always hold. We particularly expect this to be true in cases where we are unable to write down the equations of motion for Heisenberg fields, as, for example, the charge-conserving hadronic bremsstrahlung model. As a matter of fact, we believe that it is impossible to derive the S_2'' matrix from some Lagrangian (or Hamiltonian) density operator (Šoln, 1974a, b).

As far as the production unitarity is concerned, its notion is not dramatically different from the notion of usual unitarity. The difference between them is that the usual unitarity assumes every state to be accessible as an initial state at $t \to -\infty$, while what we call the production unitarity assumes only a limited number of states (selected by some projection operator P) to be accessible as initial states at $t \to -\infty$. One can easily see that hadronic bremsstrahlung models demand by their construction only primary particles to be in the initial state. Thus, for hadronic bremsstrahlung models we will always have a vacuum as the initial state for secondary particles, and consequently the S_2 matrix need only obey the production unitarity. We feel that the main reason the charge-conserving bremsstrahlung model is capable of conserving charge and parity is because the S_2'' matrix need obey only the production unitarity condition instead of the usual unitarity condition, as is the case with the ordinary hadronic bremsstrahlung model.

The usual causality conditions (3.4) and (3.5) rely heavily on the ordinary unitarity condition $S^+S = SS^+ = 1$. Both causality conditions, although elegant, appear to be too stringent, for, on the one hand, they are not compatible with our production unitarity from Section 2 and, on the other hand, they impose conditions at very small distances, which is far beyond what we can learn from experience. In fact, consistent with (3.4) one can also write down a "natural" causality property of quantum field theory: the vanishing of field commutators at spacelike-separated points ($x \sim y$) (see, for example, Horvath, 1973)

$$[\phi(x), \phi(y)] = 0 \tag{4.1}$$

where ϕ is the Heisenberg field operator. However, this commutator causality not only imposes precise conditions at infinitely small distances (beyond experimental verification) but, being very stringent, may also lead to some mathematical inconsistencies (Stapp, 1974).

As far as the ordinary hadronic bremsstrahlung model (1.7) is concerned, commutator causality (4.1) can be satisfied, for it is easy to construct the Heisenberg field associated with secondary pions as long as we do not restrict the number of initial states. However, for the case of the charge-conserving hadronic bremsstrahlung model (1.8) such a commutator causality cannot be accommodated, for here we must have restrictions on the initial states and construction of a Heisenberg field is simply not possible. This can also be seen from the fact that there is no Lagrangian from which the $S_2''(g)$ matrix would follow (Šoln, 1974a, b).

It is interesting to note that the production causality requires an integral that is performed exclusively off the pion mass-shell in momentum variable [see relation (3.13)]. An evaluation of this integral is required only because of the production causality. Consequently, it is only because of the production causality that we are required to specify the pion production amplitudes for the pion momenta off the pion mass shell [$\tilde{\omega}(k)$ appearing in (3.13) is essentially the production amplitude of a single charged pion]. This one could consider to be a weakness of the production causality for it demands a knowledge of pion production amplitudes at pion momenta not accessible by experiments. However, this weakness can be easily cured by choosing $\omega(x)$, either real or purely imaginary, for now the behavior of $\tilde{\omega}(k)$ for $k^2 \neq -\mu_\pi^2$ becomes irrelevant [see the discussion after relation (3.13)]. Such simple cures are not possible for weaknesses of causality conditions such as (4.1) that impose precise conditions at infinitely small distances. The weakness of (4.1) is in the fact that it implies the analytic properties for the scattering amplitude only outside the physical region (Stapp, 1974).

The surprising result from Section 3 is that for a charge-conserving hadronic bremsstrahlung model we do not necessarily have charge and parity conservation locally (i.e., at every instant in the course of interaction), while, of course, both charge and parity are conserved in the whole space-time interaction region, as they should be (the ordinary hadronic bremsstrahlung model conserves charge and parity neither locally nor in the whole space-time interaction region). One of the reasons why we do not have local charge and parity conservation is because we do not have a causality condition in the form $[\phi(x), \phi(x')] = 0$ for $x \sim x'$ [$\phi(x)$ being a Heisenberg field operator]. That is, the local conservation laws will rely heavily on the vanishing of field commutators for spacelike-separated points. However, as mentioned already, to demand $[\phi(x), \phi(x')] = 0$ for $x \sim x'$ goes beyond what experience tells us. Consequently to demand that the conservation laws hold at every instant of strong interactions, instead of only for the whole interaction which lasts about 10^{-23} sec, also goes beyond what experience tells us.

Acknowledgments

The author is grateful to Professor A. Donnachie and Dr. G. Shaw for the hospitality extended to him at Daresbury Laboratory.

References

Bogoliubov, N. N., and Shirkov, D. V. (1959). *Introduction to the Theory of Quantized Fields* (Interscience Publishers, Inc., New York).
Gemmel, H., and Kastrup, H. A. (1969). *Nuclear Physics B*, **14**, 566.
Horvath, Z. (1973). *Acta Physica Hungaricae*, **34**, 281.
Šoln, J. (1965). *Nuovo Cimento*, **37**, 122.
Šoln, J. (1973). *Physical Review D*, **8**, 2340.
Šoln, J. (1974a). *Physical Review D*, **9**, 3161.
Šoln, J. (1974b). *Nuovo Cimento*, **24A**, 333.
Šoln, J. (1975). *Journal of Physics G*, **1**, 709.
Stapp, H. P. (1974). In *Causality and Physical Theories*, AIP Conference Proceedings No. 16 (AIP, New York).